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VTOL TRANSPORT AIRCRAFT

Comparative Study

PROPELLER AERODYNAMICS
VERTOL REPORT NO. R-77

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Comparative Study of Various Types of
VTOL Transport Aircraft

PROPELLER AERODYNAMICS REPORT R-77

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I. SUMMARY

The vortex theory of propellers is developed in a manner suitable for the analysis of propellers for tilt-wing VTOL aircraft. Expressions defining the optimum rotor and the optimum propeller are developed. However, it is shown that a single design cannot be made which will satisfy simultaneously both optimums. From the results of computations performed with an automatic digital computer, it is concluded that in order to obtain good performance from a single design acting as both a rotor and a propeller, the propeller should be designed to operate at a high advance ratio in the airplane state. In addition, depending upon the blade solidity, the design of the propeller, with regard to pitch distribution and planform, should favor operation as a propeller rather than as a rotor.

II. NOTATION

a	$b/\sqrt{2\, C_T}$
a_o	$d\, C_l \,/\, d\alpha$
$a_{o_{inc}}$	a_o for incompressible flow
b	number of blades
c	section chord
C_d	section drag coefficient
C_l	section lift coefficient
$C_{l_{avg}}$	average lift coefficient
C_p	power coefficient = $P/\rho V_T{}^3 \pi R^2$
C_t	thrust coefficient = $T/\rho\, V_T{}^2 \pi R^2$
D	propeller diameter
F	averaging factor for induced velocity
F_Q	torque force
k	blade taper
M	figure of merit; also, Mach number
M_{cr}	critical Mach number
p	blade pitch
P	power
P_i	induced power
P_p	profile power
r	station radius
r_h	hub radius
R	propeller radius
T	thrust
V	advance velocity

V_e	resultant velocity (see Figure 4)
V_r	resultant velocity (see Figure 4)
V_t	tip velocity due to rotation only
w	propeller induced velocity
w_a	axial component of w
w_t	tangential component of w
w_o	impact velocity (see Figure 4)
w_o'	w_o/V_T
x	r/R
x_h	r_h/R
α	angle of attack
α_i	induced angle of attack
β	blade pitch angle
$\Delta\beta$	change in
γ	$C_{T_{opt}}/C_T$
Γ	bound circulation
ϵ	C_d/C_l
ω	angular velocity
ϕ	resultant flow angle
ϕ_T	ϕ for $X = 1$
σ	solidity = $bc/\pi r$
σ_o	solidity for $X = 0$
η	propeller efficiency = $C_T\lambda/C_p$
η_o	design efficiency for rotor
λ	V/V_T

subscript "opt" denotes optimum

III. INTRODUCTION

There are many factors to be considered in the design of rotors for VTOL aircraft. This present study is limited to the aerodynamic considerations.

It is fundamental that any device designed to perform a number of functions is usually never as efficient in performing any given one of those functions as a device designed specifically for that particular function. Similarly, it is to be expected that a rotor for a VTOL aircraft will not be as efficient a rotor as a helicopter rotor or as efficient of a propeller as an airplane propeller. The main difficulty in the design of a VTOL rotor is the fact that as a rotor it is loaded relatively heavy, while as a propeller it is lightly loaded. A compromise must be reached between these two states of operation.

There are several approaches which can be taken to this problem. First, the optimum designs as a rotor and as a propeller can both be investigated to determine the differences between the two. Next, the optimum rotor can be analyzed as a propeller and inversely the optimum propeller can be analyzed as a rotor. Finally, arbitrary designs can be analyzed as both a rotor and a propeller.

Other artificial means, such as retractable blades, boundary layer control or a blade with the ability to vary its twist are possible answers to satisfying the two regimes of operation. However, these are beyond the scope of this presentation.

In this development, the following problems are considered. First, the optimum twist is calculated for a rotor in hovering having a given planform and tip speed. This is then followed by the general investigation of the optimum combination of blade parameters for a rotor. These same analyses are then performed for a propeller. Finally, the analysis of a given rotor or propeller is made.

It will be seen that the optimum combination of blade parameters for a rotor and a propeller are appreciably different. In view of this conclusion, attention was given more to the analysis of constant pitch rotors performing as both rotors and propellers. A program was set up on a digital computer to study the effects of varying pertinent blade parameters. The results of these calculations are presented and discussed in the conclusions of this study.

It should be emphasized, and the reader cautioned to the fact, that this report is not intended to be a VTOL propeller design handbook. Its intended purpose is to investigate, in a broad sense, the relative merits of different design philosophies of propellers for tilt-wing, VTOL aircraft. The calculated performances presented here in comparison with measured results will probably prove somewhat optimistic. This is due to the fact a very clean airfoil section is assumed in the calculations and, in addition, the effect of a hub is ignored with all numerical integrations being carried into the axis of rotation.

IV. DEVELOPMENT OF THEORY

A. The Optimum Rotor for Hovering

The following development is based on the vortex theory of propellers as presented in references (1) and (2). A blade element in hovering is shown in Figure 1. In this figure the following quantities are defined as:

Γ = bound circulation

F_Q = torque force

T = thrust

w = induced velocity

w_a = axial component of w

w_t = tangential component of w

w_o = ficticious "impact" velocity

V_e = resultant velocity

r = section radius

x = r/R

R = rotor radius

V_t = tip velocity due to rotation

The Betz condition for the optimum propeller is well known and states that the impact velocity, w_o, must be a constant for minimum induced power loss. In addition to this condition, the induced velocity, w, must be approximately normal to the resultant velocity V_e. This condition of normality can be shown to hold exactly only in the ultimate wake where the induced velocity has increased to twice its value at the plane of the rotor.

From the geometry of Figure 1, it follows that:

(1)

$$\tan \phi = \frac{w_0}{\chi V_T} = \frac{w_a}{\chi V_T - w_t} = \frac{w_t}{w_a} = \frac{w_0'}{\chi} \qquad (a)$$

$$w_a = w_0 \cos^2 \phi \qquad (b)$$

$$w_t = w_0 \cos \phi \sin \phi \qquad (c)$$

$$V_e = \chi V_T \cos \phi \qquad (d)$$

where: $w_0' = \dfrac{w_0}{V_T}$

the Kutta-Joukowski law states that:

$$F = \rho \bar{V} \times \bar{\Gamma} \qquad (2)$$

thus, if b is the number of blades:

$$dT = b\rho \left(\chi V_T - w_t\right) \Gamma \qquad (a)$$

$$dF_Q = b\rho\, w_a \Gamma \, dr \qquad (b)$$

The circulation Γ and the tangential component of induced velocity can be related by a factor, F.

$$b \Gamma = 4\pi r F w_t \qquad (3)$$

The factor, F, has been calculated approximately by Prandtl to be:

$$F = \frac{2}{\pi} \cos^{-1} \exp \left[\frac{-b\,(1-\chi)}{2 \sin \phi_T}\right] \qquad (4)$$

where: $\phi_T = \phi$ of $\chi = 1$

Although this is an approximation to the more exact Goldstein's factor, it should agree closely with Goldstein's factor for rotors.

If equations (1) and (3) are substituted into (2) then the thrust and power coefficients can be calculated as:

(5)

$$C_T = 4 w_o'^2 \int_{x_h}^{1} F x \cos^4 \phi \, dx \qquad \text{(a)}$$

$$C_P = 4 w_o'^3 \int_{x_h}^{1} F x \cos^4 \phi \, dx \qquad \text{(b)}$$

where: $C_T = T/\rho \pi R^2 V_T^2$

$C_P = P/\rho \pi R^2 V_T^3$

P = power

x_h = hub radius/R

Equations (5 a and b) neglect any profile drag losses. If these are included, then the thrust and power coefficients become:

(6)

$$C_T = 4 w_o'^2 \int_{x_h}^{1} F x \cos^4 \phi (1 - \epsilon \tan \phi) dx \qquad \text{(a)}$$

$$C_P = 4 w_o'^3 \int_{x_h}^{1} F x \cos^4 \phi (1 + \epsilon \cot \phi) dx \qquad \text{(b)}$$

where: $\epsilon = C_d/C_l$

C_d = profile drag coefficient

C_l = section lift coefficient

It is often convenient to consider a rotor in terms of an average lift coefficient. This can be calculated by ignoring the induced velocity in calculating the thrust.

$$T = \frac{b}{2}\rho\int_{r_h}^{R}(x V_T)^2 c\, C_\ell\, dr$$

$$= \frac{b}{2}\rho\, C_{\ell_{avg}}\int_{r_h}^{R}(x V_T)^2 c\, dr \qquad (7)$$

or $$C_{l_{avg}} = \int_{x_h}^{1} x^2 \frac{c}{R} C_\ell\, dx \Big/ \int_{x_h}^{1} x^2 \frac{c}{R}\, dx$$

where: c = section chord

This can also be written in terms of the thrust coefficient.

$$C_{l_{avg}} = \frac{2\pi C_T}{b\int_{x_h}^{1} x^2 \frac{c}{R}\, dx} \qquad (8)$$

For a uniformly tapered blade where the chord is given by:

$$\frac{c}{R} = \frac{c_o}{R}\left(1 - k x\right)$$

the average lift coefficient becomes:

$$C_{l_{avg}} = \frac{6\, C_T}{\sigma_o\, f(x_h, k)} \qquad (9)$$

where: $\sigma_o = b c_o / \pi R$

$f(x_h, k) = 1 - x_h^3 - \frac{3k}{4}\left(1 - x_h^4\right)$

The optimum distribution of $\frac{cC_l}{R}$ can be determined from:

$$\frac{c\, C_\ell\, V_e}{R} = \Gamma \qquad (10)$$

or substituting for Γ and V_e from equations (1) and (3) gives:

$$\frac{c\, C_\ell}{R} = \frac{8\pi F}{b}\, w_o'\, \sin\phi \qquad (11)$$

For a uniformly tapered blade, the lift coefficient becomes:

$$C_{l_{avg}} = \frac{8 F w_o' \sin \phi}{\sigma_o (1 - k x)} \qquad (12)$$

B. Application of Equations to Design of Rotor

It will be assumed that the blade airfoil section, planform, and tip speed are known from other considerations. Because of the profile drag losses, it is difficult to determine prior to the detailed calculations, the exact value of w_o' necessary to give the desired thrust coefficient. However, as a start, in equation 6(a) the factor F can be taken to be 1.0 and $\epsilon = 0$ so that approximately:

$$C_T \cong 2 w_o'^2$$

$$\text{or:} \quad w_o' \cong \sqrt{C_T/2} \qquad (13)$$

The procedure for applying the equations which have been developed thus far is perhaps better presented in steps.

1. Calculate w_o' from equation (13)
2. Calculate ϕ from equation (1a)
3. Calculate F from equation (4)
4. Calculate CC_l from equation (11)
5. Knowing the chord distribution, the lift coefficients can be obtained from step 4 and then C_d from the airfoil section characteristics.
6. Calculate C_T and C_P from equation (6)
7. Steps 1 thru 6 are repeated until the desired C_T is obtained.
8. Calculate blade pitch angles from -

$$\beta = \phi + \frac{C_l}{a_o} \tag{14}$$

where:
$$a_o = \frac{d\,C_l}{d\alpha}$$

C. Selection of Optimum Rotor Parameters

The choice of the rotor solidity and thrust coefficient can be made on the basis of aerodynamic considerations. In so doing, the profile drag is the governing factor. In the absence of profile drag, the optimum rotor for supporting a given weight is obviously the largest one which can be tolerated. This follows, since from equation (6) for C_d = 0 and F = 1.0:

$$C_p = 2\left(\frac{C_T}{2}\right)^{3/2}$$

or

$$P = \frac{T^{3/2}}{\sqrt{2\rho\pi R^2}}$$

However, when the profile drag is included, there will be a point beyond which the increased loss in profile power will offset any decrease in induced power gained by increasing the radius.

To determine this optimum radius, the assumption is made that the angle ϕ is small so that cos ϕ = 1. Then equation (5b) for the power can be written as:

$$P = 4\pi\rho R^2 V_T^3 w_o'^3 \int_{x_h}^{1} x F\, dx + 4\pi\rho R^2 V_T^3 w_o'^2 \int_{x_h}^{1} x^2 F \epsilon\, dx$$

It can be shown that, to the approximation that ϕ is small:

$$F = \frac{\sigma_x C_l x}{8 w_o'^2}$$

where: $\sigma_x = \dfrac{bc}{\pi R}$

thus: $$P = 4\pi\rho R^2 V_T^3 w_o'^3 \int_{x_h}^{1} x F\, dx + \frac{\pi}{2}\rho R^2 V_T^3 \int_{x_h}^{1} \sigma_x x^3 C_d\, dx$$

If equation (5a) is substituted for w_o' then:

$$P = \left[\frac{T^{3/2}}{\left(4\pi\rho \int x F\, dx\right)^{1/2}}\right]\frac{1}{R} + \left[\frac{\pi\rho V_T^3 \int x^3 \sigma_x C_d\, dx}{2}\right] R^2$$

This is of the form:

$$P = \frac{C_1}{R} + C_2 R^2$$

For a constant tip speed, T, and σ, the power will be a minimum when -

$$\frac{dp}{dR} = 0 = -\frac{C_1}{R^2} + 2C_2 R$$

or

$$R^3_{opt} = C_1 / 2C_2$$

Notice that:

$$\frac{C_1}{R_{opt}} = 2\, C_2\, R_{opt}^2$$

That is, for minimum power, the induced power should be twice the profile power, a result which has been derived previously in reference (3). The expression for the optimum radius can be written as -

$$R_{opt}^3 = \frac{\left(\frac{T}{\rho V_T^2}\right)^{3/2}}{\pi \sigma_o \left[\int_0^1 x^3 (1 - k x) C_d \, dx\right] \sqrt{4\pi \int_0^1 x F \, dx}} \tag{15}$$

Let:

$$\int_0^1 x^3 (1 - k x)\, C_d \, dx = I_2$$

Equation (15) can be rewritten giving a relationship between the thrust coefficient and the solidity for minimum power.

$$\sigma_{o_{opt}} = \frac{C_{T_{opt}}^{3/2}}{2 I_2 \sqrt{\int_0^1 x F \, dx}} \tag{16}$$

It can be shown by expanding F in a series that:

$$\int_0^1 x F \, dx \cong \frac{1}{2} - \frac{19}{9\pi a}$$

where: a $= \dfrac{b}{\sqrt{2 C_T}}$

If, in addition, it is assumed that the drag coefficient is a constant, then the following relationship can be obtained.

$$(1-0.8k)C_d \sigma_{o_{opt}} = \frac{2C_T^{3/2}}{\sqrt{0.5-\frac{0.672}{a}}} \qquad (17)$$

If equation (9) for $C_{l_{avg}}$ is substituted for σ_o in equation (17), then it will be found that -

$$\frac{(1-0.75k)}{(1-0.8k)}\frac{C_{l_{avg}}}{C_d} = \sqrt[3]{\frac{0.5-\frac{0.672}{a}}{C_T}} \qquad (18)$$

Equations (17) and (18) are plotted as a function of the thrust coefficient for different numbers of blades in Figure 2.

Before going to the optimum propeller, consider the implications of the foregoing development. The figure of merit of a rotor is defined as the ratio of the ideal induced power required by a rotor to hover to the actual power required.

$$M = \frac{P_i}{P_i + P_p}$$

where: P_i = induced power

P_p = profile power

Most texts on helicopters, for example reference (4), state that M, for a good rotor, should be about 0.75. For the ideal rotor M = 1, while, according to reference (4), M = 0.5 is poor for a rotor. But now consider M for the optimum rotor including profile drag. For this case it was just shown that the induced power should equal twice the profile power. Thus

$$M = \frac{2P_p}{2P_p + P_p} = \frac{2}{3}$$

The value of M = 0.667 is rather unexpected and much lower than usually thought acceptable, and yet this is the value for a rotor requiring the least total power to produce a given thrust. Thus, the usual figure of merit for a rotor can be misleading and not a reliable basis for comparing different rotor designs.

A different standard for rotors in hovering is therefore proposed. Instead of using the ideal induced power as a basis for comparison it is proposed to use instead the power which would be required by the optimum rotor. For want of a better name, this new "figure of merit" will be called "design efficiency" and denoted by η_0.

$$\eta_0 = \frac{P_{opt}}{P} \tag{19}$$

Now $P \cong T w_0 + \frac{\rho}{8} b c R V_T^3 C_d$

$$P_{opt} = \frac{3}{2} T w_{opt}$$

so that:

$$\eta_0 = \frac{w_{opt}}{\frac{2 w_0}{3} + \frac{\rho b c R V_T^3 C_d}{12 T}}$$

or

$$\eta_0 = \frac{\frac{w_{opt}}{w_0}}{\frac{2}{3} + \frac{\rho b c R V_T^3 C_d}{12 T w_0}}$$

Now:

$$C_d \sigma_{opt} = 2\sqrt{2}\;\lambda_{opt}$$

and: $$w_o = V_T \sqrt{\frac{C_T}{2}}$$

so that:
$$\eta_o = \frac{\gamma^{1/2}}{\frac{2}{3} + \frac{\gamma^{3/2}}{2}} \tag{20}$$

where:
$$\gamma = C_{T_{opt}} / C_T$$

Equation (20) is plotted as a function of γ in Figure 3. From this figure it can be seen that a rotor can be operating 20% off of the optimum C_T with only a 1% penalty in power.

D. <u>The Optimum Propeller</u>

A development will now be undertaken for the propeller similar to that which was just presented for the rotor. Consider the propeller blade element shown in Figure 4. In this development, the propeller will be assumed to be lightly loaded. Hence, the angle α_i, the induced angle of attack, can be treated as a small angle. In the same manner as for the rotor, the thrust and torque force per blade can be written as:

(21)

$$dT = b\rho\Gamma(xV_T - w_t)\,dr \qquad \text{(a)}$$

$$dF = b\rho\Gamma(V + w_a)\,dr \qquad \text{(b)}$$

where: V = inflow velocity

Again, for the optimum propeller, the Betz condition holds that w_o, the impact velocity, must be constant. Thus:

V

$$\tan\phi = \frac{V + w_o}{x V_T} = \frac{w_t}{w_a} = \frac{V + w_a}{x V_T - w_t} \qquad (22)$$

Also:

$$b\Gamma = 4\pi r F w_t$$

The expressions for thrust and torque force therefore become:

(23)

$$dT = \frac{4\pi\rho r F}{b} w_a (V + w_a)\, dr \qquad (a)$$

$$dF_Q = dT \left(\frac{V + w_a}{x V_T}\right) \qquad (b)$$

or

$$dP = (V + w_o)\, dT \qquad (c)$$

The ideal efficiency (neglecting profile drag) can be seen immediately to be,

$$\eta = \frac{TV}{P} = \frac{1}{1 + w_o/V}$$

Including the profile drag, the thrust and power coefficients become:

(24)

$$C_T = 4 w_o' \int_{x_h}^{1} F x \cos^2\phi (\lambda + w_o' \cos^2\phi)(1 - \epsilon \tan\phi)\, dx \qquad (a)$$

$$C_P = 4 w_o' \int_{x_h}^{1} (\lambda + w_o') F x \cos^2\phi (\lambda + w_o' \cos^2\phi)(1 + \epsilon \cot\phi)\, dx \qquad (b)$$

where: $\lambda = V/V_T$

$w_o' = w_o/V_T$

$\epsilon = C_d/C_l$

As an approximation, w_0' and C_T are related by -

$$w_o' = \frac{1}{2}\left[-\lambda + \sqrt{\lambda^2 + 2\, C_T}\right] \tag{25}$$

E. Selection of Optimum Propeller Parameters

The problem of choosing the solidity and radius for an optimum propeller is now considered. The power can be written as:

$$dP = (V + w_o)\,dT + \frac{b}{2}\rho V_e^2 c\, C_d \cos\phi\, \chi\, V_T\, R\, d\chi$$

If the last term is called P_o, then

$$dP_o = \frac{b}{2}\rho V_e^2 c\, C_d \cos\phi\, \chi\, V_T\, R\, d\chi$$

$$= \frac{b}{2}\rho V_e^2 c\, C_d \left(\frac{\chi V_T - w_t}{V_e}\right)\chi\, V_T\, R\, d\chi$$

If equation (19a) is substituted into the above then -

$$dP_o = \frac{V_e}{2}\;\frac{c\, C_d\, dT}{\Gamma}$$

$$= \frac{\sigma_x V_e V_T C_d\, dT}{8\chi F w_t}$$

but $V_e = \dfrac{w}{\tan\alpha_i} = \dfrac{w_t}{\sin\phi\,\tan\alpha_i}$

therefore:

$$dP_o = \frac{V_T\, \sigma_x\, C_d\, dT\, \cot\alpha_i}{8\chi F \sin\phi}$$

The total power is thus:

$$dP = V\,dT + \frac{V_r \sin\alpha_i}{\cos\phi} + \frac{V_T\, \sigma_x\, C_d\, dT\, \cot\alpha_i}{8\chi F \sin\phi}$$

If the elemental thrust is held constant, the elemental power will be a minimum when

$$\frac{dP}{d\alpha_i} = 0 = \frac{V_r \cos \alpha_{i_{opt}}}{\cos \phi} - \frac{\sigma_x C_d V_T}{8 \chi F \sin \phi} \frac{1}{\sin^2 \alpha_{i_{opt}}}$$

α_i is small so that:

$$\alpha_{i_{opt}}^2 = \frac{\sigma_x C_d V_T \cos \phi}{8 \chi F V_r \sin \phi}$$

If the value of α_i is substituted into dP_o then

$$dP_o = \frac{V_r}{\cos \phi} \alpha_{i_{opt}} dT \qquad (27)$$

but (27) is simply the induced power given by the second term in equation (26). Thus, for the optimum propeller with minimum power, the induced power should equal the profile power.

Thus:

$$P_{min} = (V + 2 w_o) T \qquad (28)$$

This result establishes an upper limit on the efficiency of a propeller with the profile drag included.

$$\eta_{max} = \frac{TV}{P_{min}} = \frac{1}{1 + 2 \, w_o/V}$$

or, since $\frac{w_o}{V}$ is given closely by

$$\frac{w_o}{V} = \frac{1}{2} \left[-1 + \sqrt{1 + \frac{2 C_T}{\lambda^2}} \right]$$

it follows that:

$$\eta_{max} = \frac{1}{\sqrt{1 + \frac{2 C_T}{\lambda^2}}} \qquad (29)$$

A relationship between $C_{T_{opt}}$, σ and λ can be obtained by writing the thrust coefficient approximately as:

$$C_T = \int_0^1 4x^2 F \lambda \alpha_i \, dx$$

and substituting from 26 for α_i.

$$C_{T_{opt}} = \sqrt{2 \lambda \sigma_0} \int_0^1 \sqrt{C_d (1-kx)} \, x^2 \sqrt{F \cos\phi} \, dx \qquad (30)$$

An approximate relation between $C_{T_{opt}}$, σ and λ can be obtained by letting $k = 0$ (no taper) and assuming $F = 1$ and C_d = constant.

$$\frac{C_{T_{opt}}}{\sqrt{C_d \sigma}} = \sqrt{2\lambda} \int_0^1 \frac{x^{5/2}}{(x^2 + \lambda^2)^{1/4}} \, dx \qquad (31)$$

The integral in equation (31) has been evaluated graphically as a function of λ. The result is presented in Figure 5. Also included in the figure are results obtained from reference (9) of experimental measurements with three-bladed variable pitch variable pitch propellers. For each collective pitch angle the value of the propeller advance ratio and thrust coefficient for maximum efficiency was chosen to give the point shown. The experimental trend is seen to be in substantial agreement with the theory.

F. Comparison of Optimum Rotor with the Optimum Propeller

The thrust coefficient of a VTOL rotor acting as a propeller can be expressed in terms of its coefficient as a rotor. If a sub "p" refers to propeller and a sub "r" to rotor, then

$$C_{T_p} = \frac{C_D}{C_L}\left(\frac{N_r}{N_p}\right)^2 C_{T_r} \tag{32}$$

where: C_D = aircraft drag coefficient

C_l = aircraft lift coefficient

N = rotational speed

The question is now posed as to whether equation (32) and Figures 2 and 5 can be satisfied simultaneously. Consider typical values of

C_{T_r} = .016

C_D/C_L = 1/12

λ = .35

From Figure 2, for C_d = 0.01, σ_{opt} = 0.58

From Figure 5, $C_T/\sqrt{C_d\sigma} = .0198$

therefore: $N_p/N_r = 0.284$

In terms of the usual rotor tip speeds, this would give a very low N_p and, hence, for this λ a very low aircraft speed. Increasing C_T does not change this result appreciably. Now choose λ = 1.0. Then $C_{T_p} = .43\sqrt{C_d\sigma}$ or C_{T_p} = .0327

therefore: $N_p/N_r = 0.221$

Again even for this relatively high λ, for the usual rotor tip speeds, this results in low aircraft speeds. Actually, the value of $C_{T_p}/\sqrt{C_d \sigma}$ for λ larger than 1 should not change much beyond the value of 0.4. Thus $\frac{N_p}{N} = 0.22$ and $\lambda \cong 5$ would be needed in order for both the rotor and propeller to be compatible with optimum requirements.

From these examples, it does not appear feasible to attempt a design which would have an optimum solidity as both a rotor and a propeller. Therefore attention will now be given to the problem of analyzing a given rotor or propeller.

G. Analysis of Given Rotor or Propeller

Reference is again made to Figure 4. Now, instead of finding the β distribution to give the optimum induced velocity distribution, the β will be given. The problem then is to find the induced velocity and hence the lift and drag coefficients for given V and V_T values. To do this, the bound circulation, Γ, is written first as:

$$\Gamma = \frac{1}{2} c \, C_l V_e$$

but Γ is also given approximately by:

$$\Gamma = \frac{4\pi r F w_t}{b}$$

Equating the two expressions for Γ gives

$$C_l = \frac{8 x F w_t}{\sigma_x V_e} \tag{33}$$

but C_l is also given by:

$$C_l = a_o (\beta - \phi) \tag{34}$$

so that
$$a_0(\beta - \phi) = \frac{8 \chi F w_t}{\sigma_x V_e}$$

From the geometry of Figure 4, it can be seen that -

$$w_t = V_e \tan \alpha_i \sin \phi \tag{35}$$

Thus:

$$a_0(\beta - \theta - \alpha_i) = \frac{8 \chi F}{\sigma_x} \tan \alpha_i \sin \phi$$

now:
$$\phi = \tan^{-1} \frac{\lambda}{\chi} + \alpha_i$$

Thus the induced angle of attack is given implicitly by:

$$a_0(\beta - \theta - \alpha_i) = \frac{8 \chi F}{\sigma_x} \tan \alpha_i \sin \left[\tan^{-1} \frac{\lambda}{\chi} + \alpha_i\right] \tag{36}$$

This equation can be solved explicitly to the approximation that α_i is small:

$$\sigma_x a_0(\beta - \theta - \alpha_i) = 8 \chi F \alpha_i (\sin \theta + \alpha_i \cos \theta)$$

Therefore:

$$\alpha_i = \frac{1}{2}\left[-B + \sqrt{B^2 + 4C}\right] \tag{37}$$

where: B $= \tan \theta + \dfrac{\sigma_x a_0}{8 \chi F \cos \theta}$

$$C = \frac{\sigma_x \, a_o \, (\beta - \theta)}{8 \chi F \cos \theta}$$

For a rotor in hovering $\lambda = 0$ so that $\theta = 0$.

For this case, the induced angle of attack becomes:

$$\alpha_i = \frac{\sigma_x \, a_o}{16 \chi F}\left[\sqrt{1 + 4\left(\frac{8 \chi F}{\sigma_x \, a_o}\right)\beta} - 1\right] \qquad (38)$$

The tip loss factor F is given by (4). The angle ϕ_T in equation (4) can be taken to be β_T, the blade pitch angle at the tip.

Thus knowing β, σ_x, λ and the airfoil section characteristics, the radial variation of α_i can be calculated. The section lift coefficients are then obtained from (34).

The thrust and power coefficients can be calculated by:

$$C_T = \frac{T}{\rho V_T^2 \pi R^2} = \frac{1}{2}\int_0^1 \sigma_x \left(\frac{V_e}{V_T}\right)^2 C_l \cos\phi \, (1 - \epsilon \tan\phi) \, d\chi \qquad (a)$$

$$C_P = \frac{P}{\rho V_T^3 \pi R^2} = \frac{1}{2}\int_0^1 \chi \, \sigma_x \left(\frac{V_e}{V_T}\right)^2 C_l \sin\phi \, (1 + \epsilon \cot\phi) \, d\chi \qquad (b)$$

The velocity (V_e/V_T), to the approximation of α_i being small, is given closely by: $\left(\frac{V_e}{V_T}\right)^2 = \lambda^2 + \chi^2$

H. Results of Digital Computer Program

An extensive program has been performed on an IBM Model 650 magnetic drum digital computer. The program considered only rotors or propellers having constant pitch; that is, with blade pitch angles given by:

$$\beta = \tan^{-1}\frac{P/D}{\pi x} + \Delta\beta \tag{40}$$

where: p = propeller pitch

$\Delta\beta$ = propeller collective pitch angle.

This particular pitch distribution was chosen when it was found to agree closely with the β distributions calculated for the optimum rotors or propellers for $\Delta\beta = 0$.

The drag coefficient variation with Mach No. was taken proportional to the fourth power of the difference between the critical Mach No. and the local operating Mach No. This is in accordance with the recommendation of references (5) and (6). Specifically, the drag coefficient was expressed as:

$$C_d = C_{d_o} + C_2\, C_l^2 \qquad M \leq M_{cr}$$

$$= C_{d_o} + C_2\, C_l^2 + K(M - M_{cr})^4 \qquad M > M_{cr}$$

The critical Mach No. is given approximately by:

$$M_{cr} = M_{cr_o} - m_1 C_l$$

where: C_2, K, and m_1 are constants of proportionality

C_l = section lift coefficient

C_{d_o} = C_d for $C_l = 0$

M_{cr_o} = critical Mach No. for $C_l = 0$

For the 12% thick series 16 airfoils which were used in this study, the various constants were determined from reference (7) as:

C_2 = 0.006

K = 200

m_1 = 0.162

M_{cr_o} = 0.757

C_{d_o} = 0.006

The slope of the section lift coefficient curve was corrected for compressibility effects by fitting the emperical results of reference (8) with the following expression:

$$a_o = a_{o_{inc}} (1 + a_4 M^4 + a_{10} M^{10})$$

The constant a_4 and a_{10} were found to be:

a_4 = 1.438

a_{10} = −4.29

$a_{o_{inc}}$ is the slope of the lift curve for incompressible flow and for a 12% thick series 16 airfoil was taken as $a_{o_{inc}} = 4.8$.

The effect of Reynold's No. was not considered in these calculations. The Reynold's No. for the test data of reference (7) was between 0.85×10^6 and 2×10^6. As long as the Reynold's number of an average propeller station, say the 0.7 radius, is of this order the results given here should be valid.

The basic results which were obtained are presented in Figures 6 through 16. Figure 6 presents the calculated thrust and power coefficients of families of rotors having constant pitch with $\Delta\beta$ = 0. These curves represent about the best performance that could be obtained with a hovering rotor since the aerodynamic loading is close to ideal and their airfoil sections are very clean. These results are all for a constant tip Mach No. of 0.75.

The effect of tip Mach No. on a hovering rotor as predicted by the present methods is shown in Figure 7. For a given thrust coefficient, the power coefficient is seen to be nearly the same for M_T = .8 as for M_T = .6, since for a given pitch-diameter ratio both the thrust and power coefficients are higher for M_T = 0.8. For a tip Mach No. of 0.9 the performance of the rotor is seen to be seriously affected. The thrust coefficient, for a given P/D, is lower than that of M_T = .8 while the power coefficient is appreciably higher.

Figures 8, 9 and 10 present the calculated thrust and power coefficient as a function of the collective pitch angle for propellers whose P/D values were chosen from Figure 6 to have average C_l values in hovering of 0.5. As noted on the figures, the performance of the propeller was calculated for three different values of the forward speed-tip speed ratio, λ .

The calculated performance of constant pitch propellers with $\Delta\beta$ = 0 for different λ values is presented in Figures 11, 12 and 13. These curves are all calculated at a constant forward

Mach No. of 0.4.

Figures 14, 15 and 16 present the calculated performance of the propellers of the previous three figures when performing as rotors. The P/D values were chosen such that, as propellers with $\Delta\beta = 0$, the thrust coefficient would be 0.005.

V. CONCLUSIONS

Although it appears difficult to state any definite rules for the selection of a VTOL propeller, there are some general observations and conclusions that can be made from the results of this study. From the analyses of the optimum rotor and the optimum propeller, it appears impossible to attain both of these optimums in a single design. However, certain design practices can be followed which should assure a good aerodynamic compromise between the states of operation.

These practices are evident from Figures 17 and 18. These figures have been constructed from the results given in Figures 6 through 16. Figure 17 presents the power required by the optimum rotor acting as a propeller to deliver a given thrust coefficient as compared with the power required by the optimum propeller to deliver the same thrust coefficient. Figure 18 presents the power required by the optimum propeller to hover at a given thrust coefficient compared with the power required by the optimum rotor.

Regardless of whether the VTOL propeller is designed as an optimum propeller or as an optimum rotor or as a compromise, one important fact is obvious from both Figures 17 and 18. The ratio, λ, of the propeller forward speed to the tip speed should be relatively high. The gain in power from increasing λ is two-fold. Not only does the ratio of power coefficients decrease with increasing λ at a constant C_t value but, in addition, for a given thrust and forward speed, the C_t value increases with increasing λ which further reduces the ratio of power coefficients. There is a limit, of course, to the value of λ above which the power required to produce a given thrust at a certain forward speed will increase. This optimum λ can be estimated from Figure 5.

Another conclusion of some significance is to be drawn from these figures. This can best be shown by considering a typical example. Assume a tilt-wing VTOL transport with the following characteristics:

gross weight	100,000#
forward velocity	400 fps
number of propellers	4
average C_1 in hovering	0.5
drag in forward flight	10,000#

The power required to hover using an optimum rotor and an optimum propeller is shown as a function of the propeller advance ratio for different values of solidity in Figure 19. The power required for forward flight for the optimum propeller and optimum rotor is given in Figure 20 as a function of λ. Now it is felt that because of weight and other considerations, the blade solidity of a VTOL aircraft will be high in comparison with the usual helicopter rotor. From Figure 19, it can be seen that over the range of values considered, the power required to hover by the propeller for solidities of 0.3 and 0.5 is at the most, only 4% higher than the power required by the optimum rotor. However, from Figure 20, the power required by the optimum rotor in forward flight is at least 10% higher than that required by the optimum propeller at the higher λ values and at the lower values of λ is more than 50% higher than for the propeller.

Thus, it would appear advisable in designing the VTOL propeller, that is in selecting the blade twist and planform, to favor the operation of the propeller in the airplane state.

VI. REFERENCES

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FIGURE 1

ROTOR BLADE ELEMENT IN HOVERING

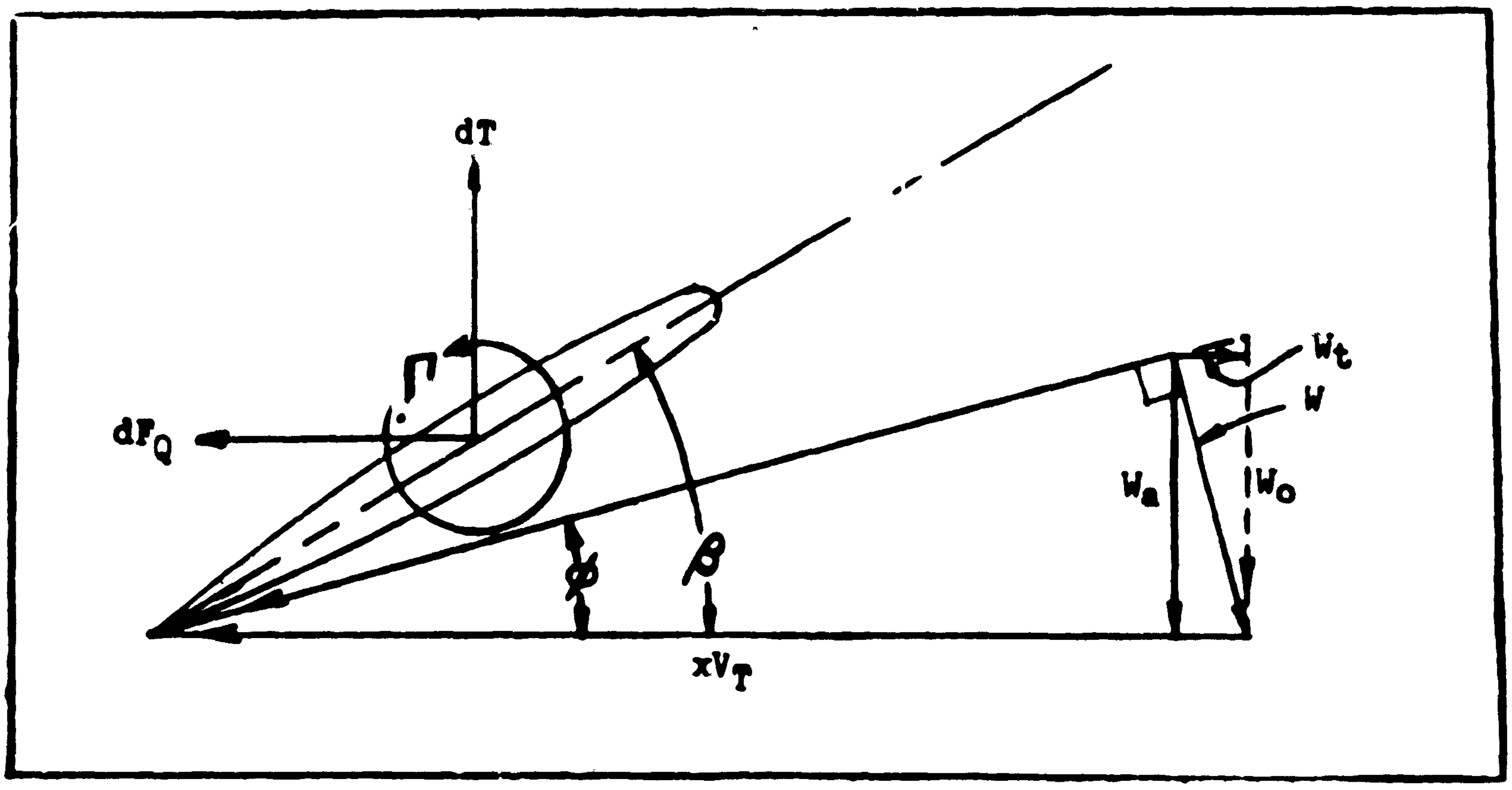
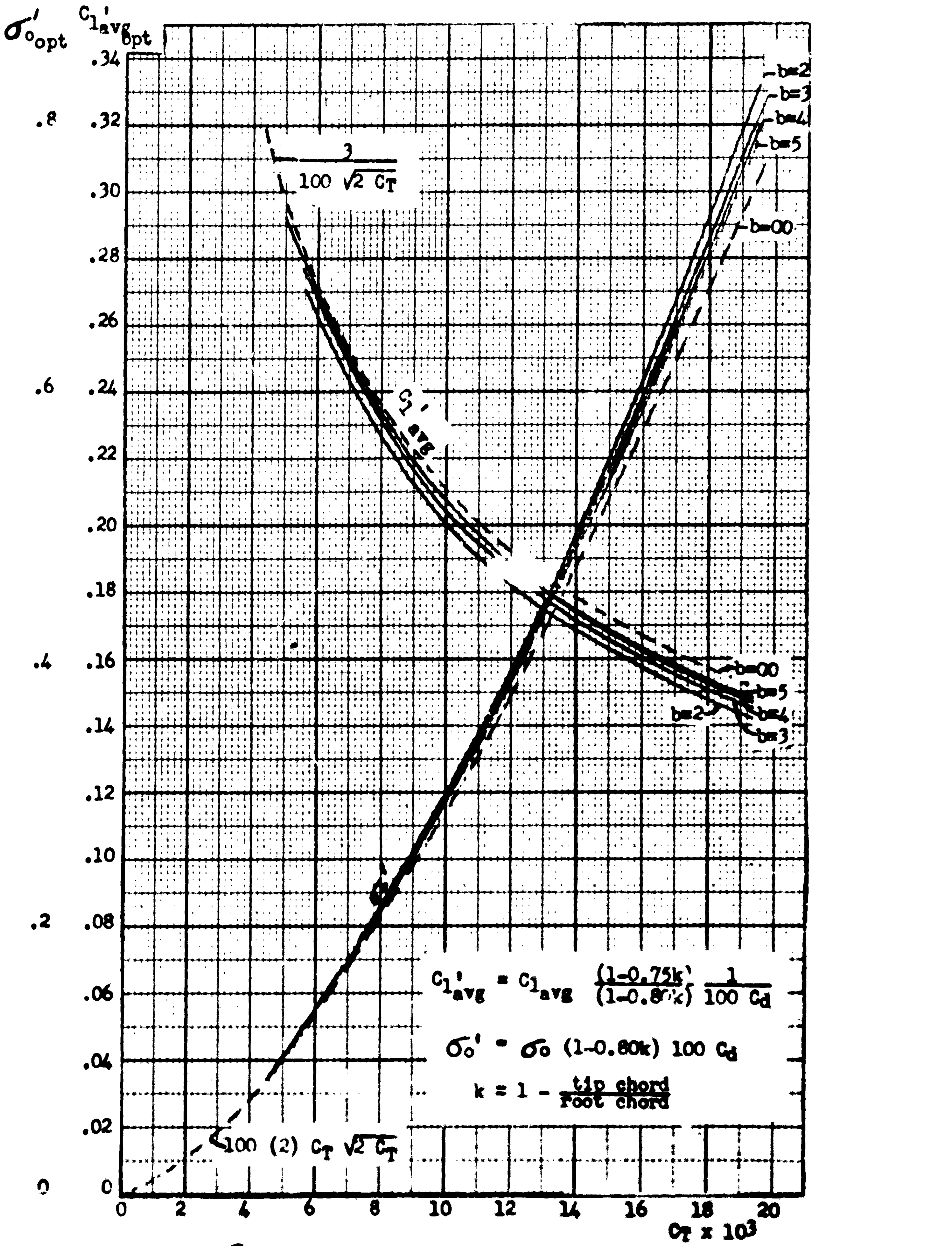


FIGURE 2

RELATIONSHIP BETWEEN OPTIMUM SOLIDITY, THRUST COEFFICIENT AND AVERAGE LIFT COEFFICIENT FOR A ROTOR



Optimum σ_o & $C_{l_{avg}}$ vs C_T for Linearly Tapered Optimum Rotor in Hovering

FIGURE 3

DESIGN EFFICIENCY

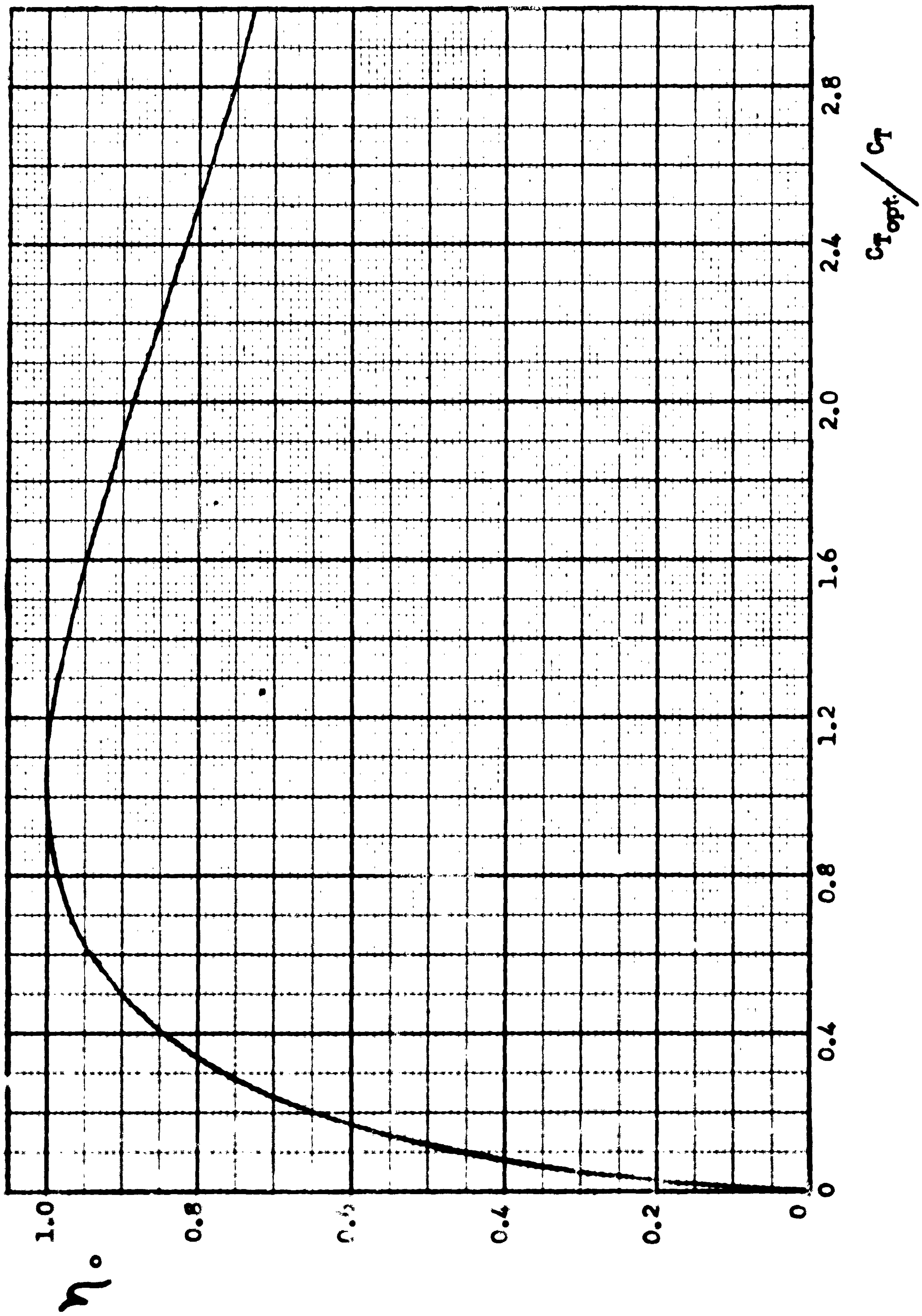


FIGURE 4

PROPELLER BLADE ELEMENT

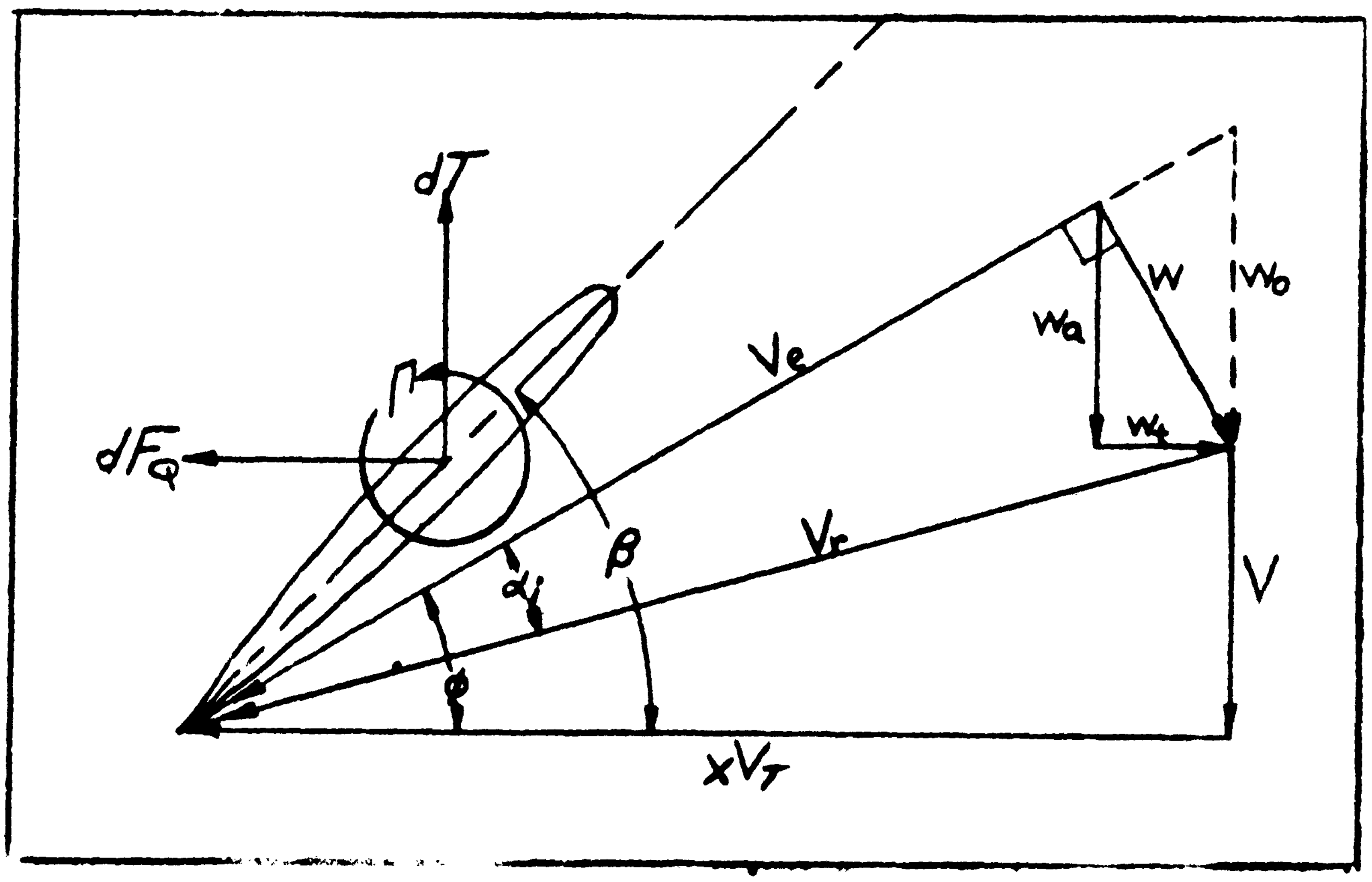


FIGURE 5

RELATIONSHIP BETWEEN OPTIMUM SOLIDITY, THRUST COEFFICIENT AND λ FOR A PROPELLER

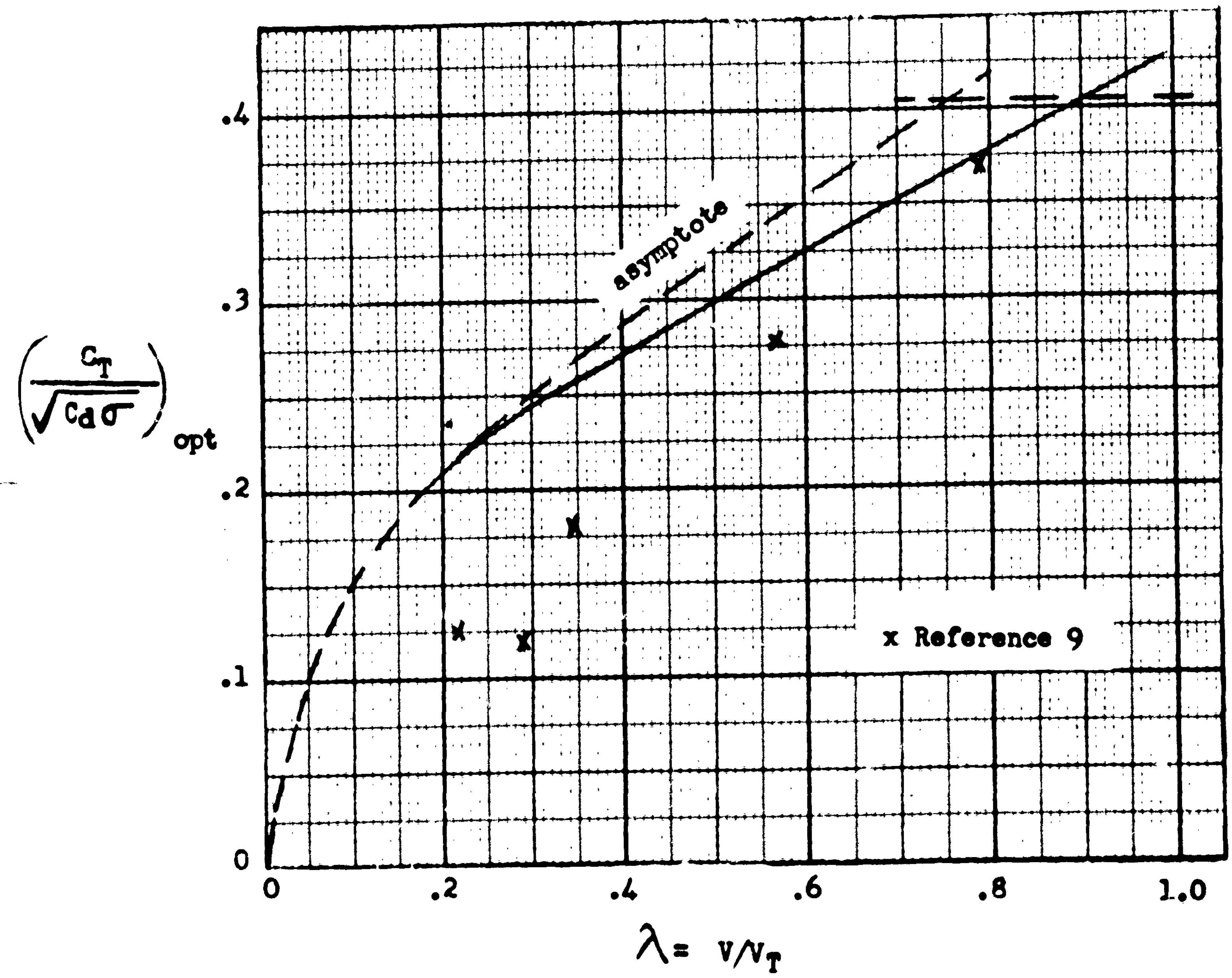
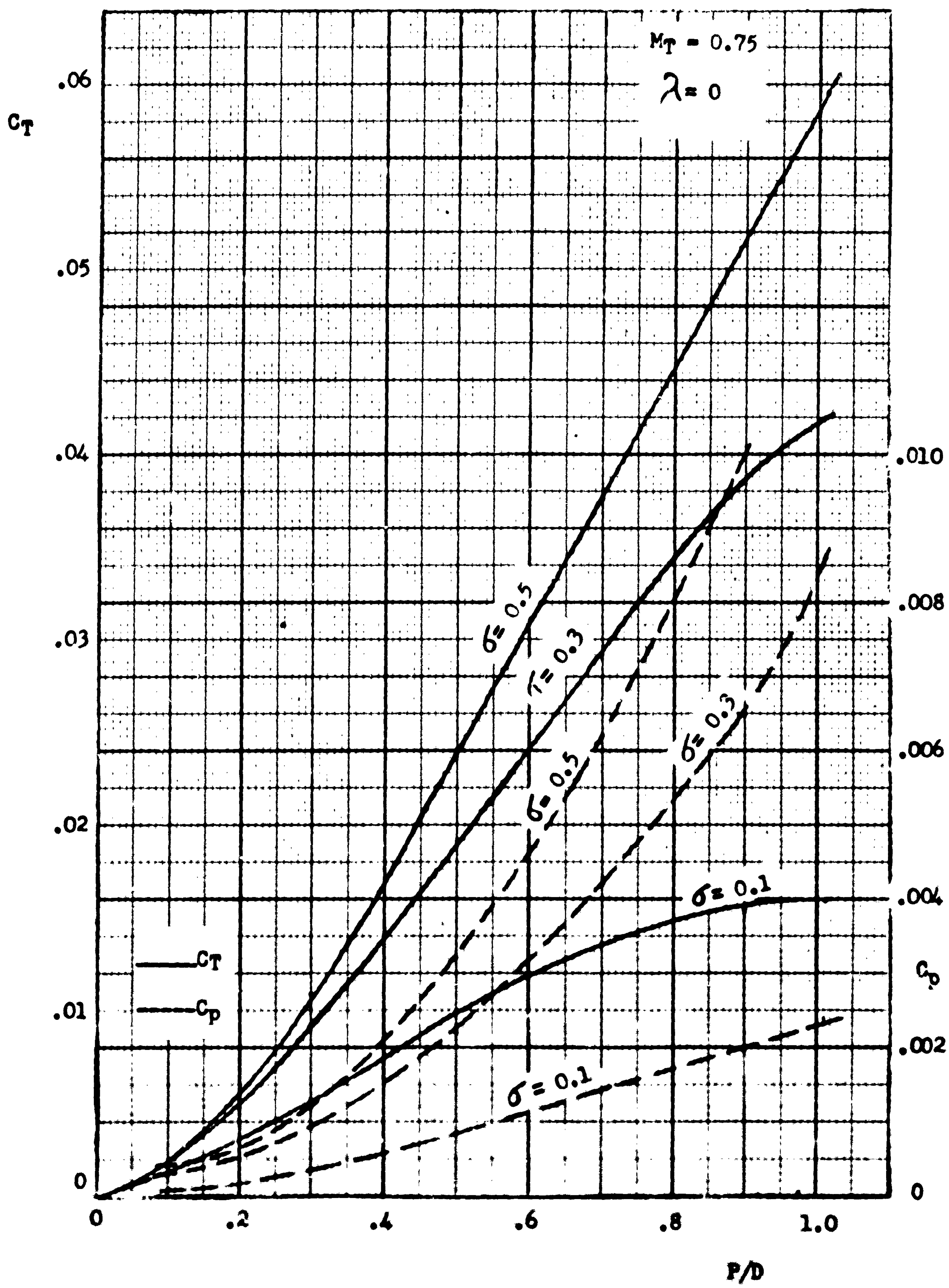


FIGURE 6

CALCULATED PERFORMANCE OF CONSTANT - PITCH ROTORS



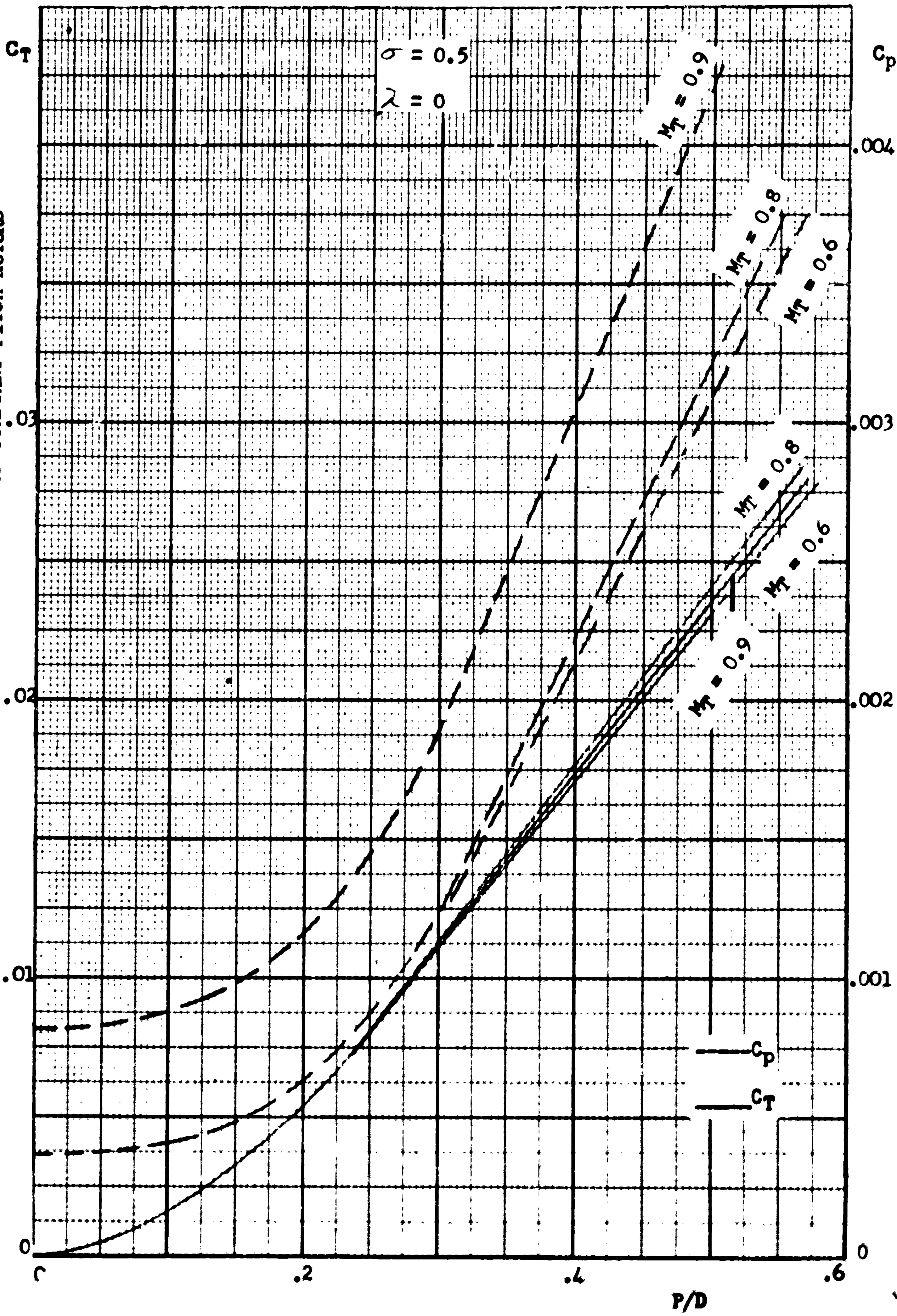


FIGURE 7

EFFECT OF MACH NUMBER ON THE PERFORMANCE OF CONSTANT PITCH ROTORS

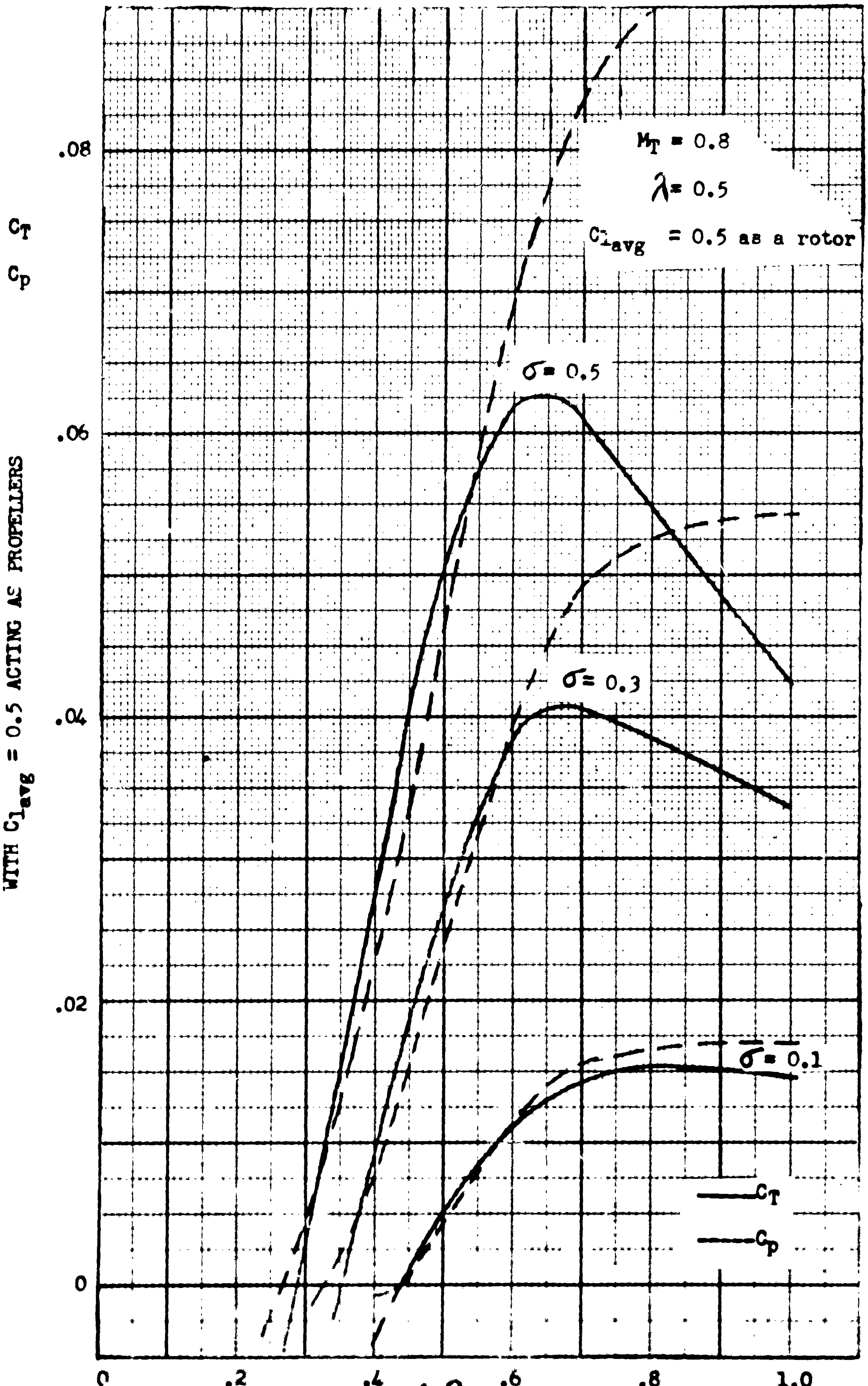


FIGURE 8

CALCULATED PERFORMANCE OF CONSTANT - PITCH ROTORS WITH $C_{l_{avg}}$ = 0.5 ACTING AS PROPELLERS

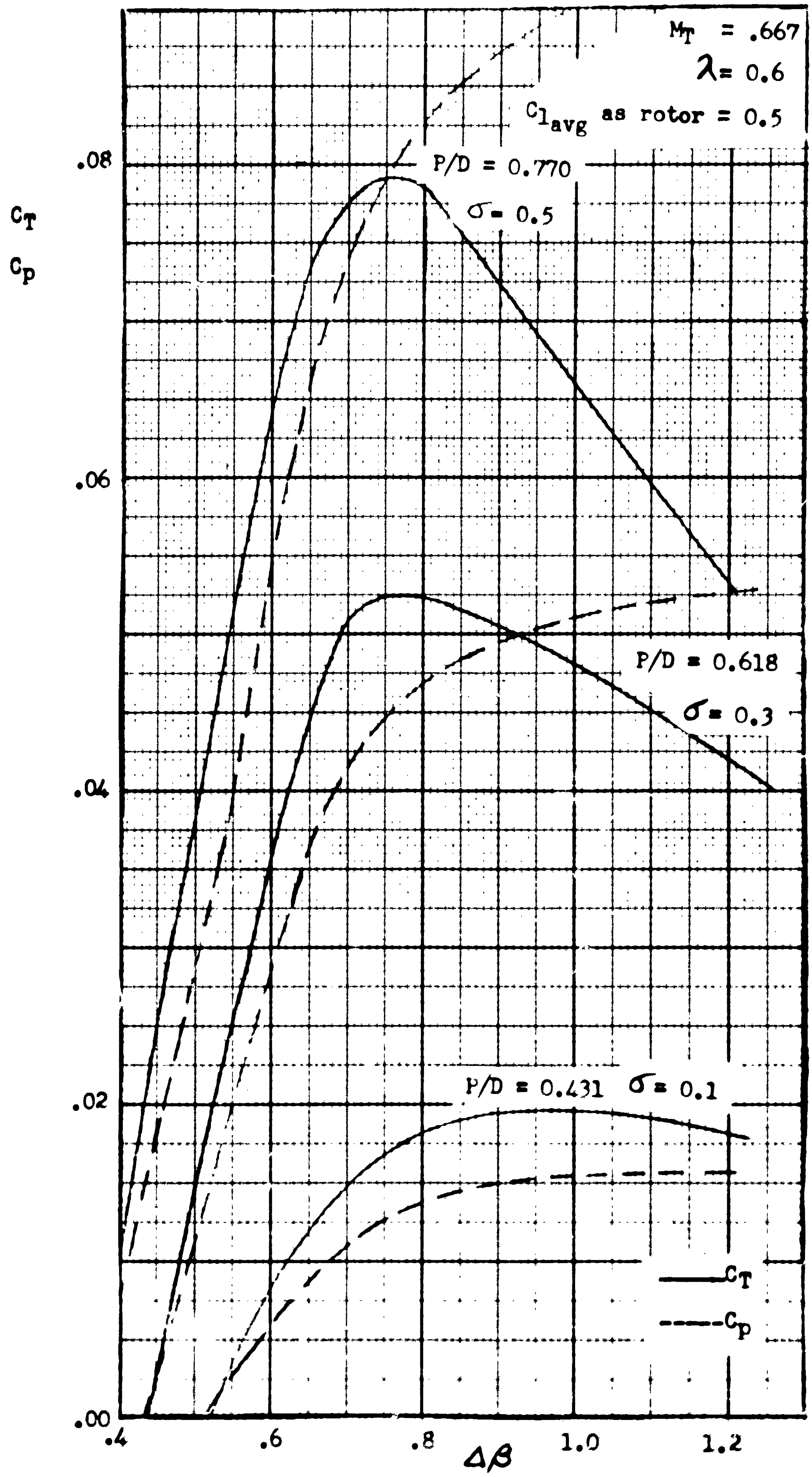


FIGURE 9

CALCULATED PERFORMANCE OF CONSTANT - PITCH ROTORS WITH $C_{l_{avg}}$ = 0.5 ACTING AS PROPELLERS

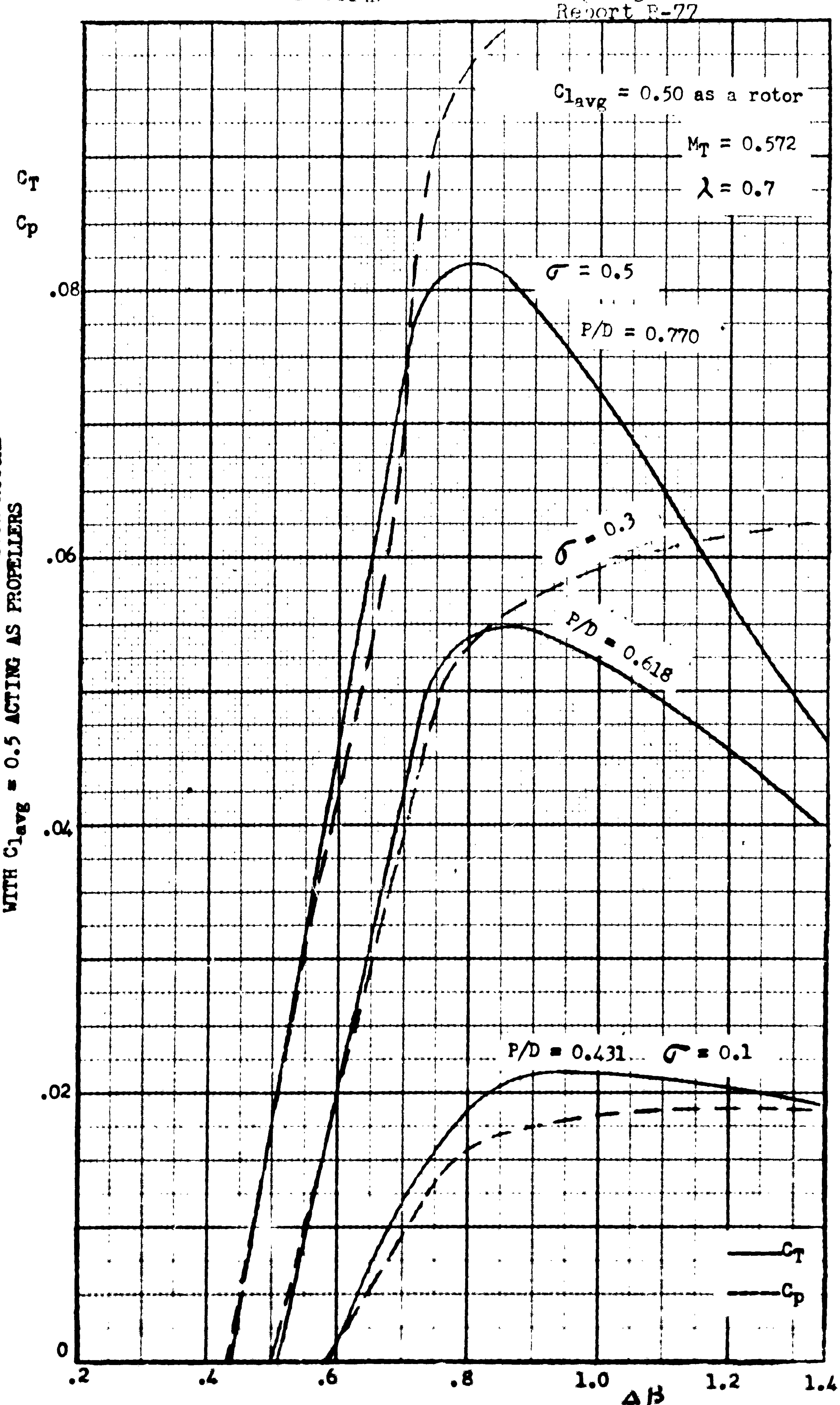


FIGURE 10

CALCULATED PERFORMANCE OF CONSTANT - PITCH ROTORS WITH $C_{l_{avg}}$ = 0.5 ACTING AS PROPELLERS

FIGURE 11

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS

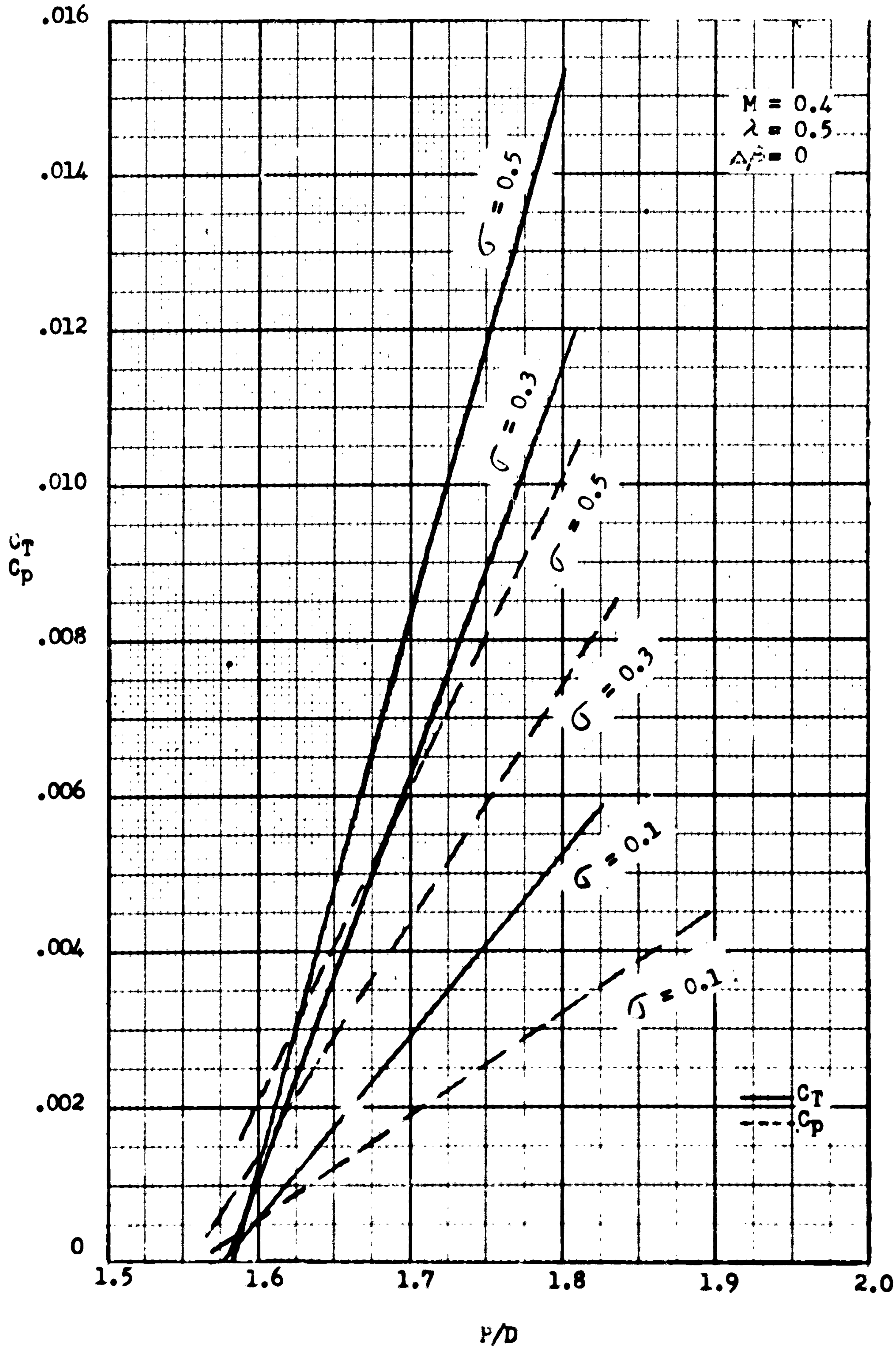


FIGURE 12

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS

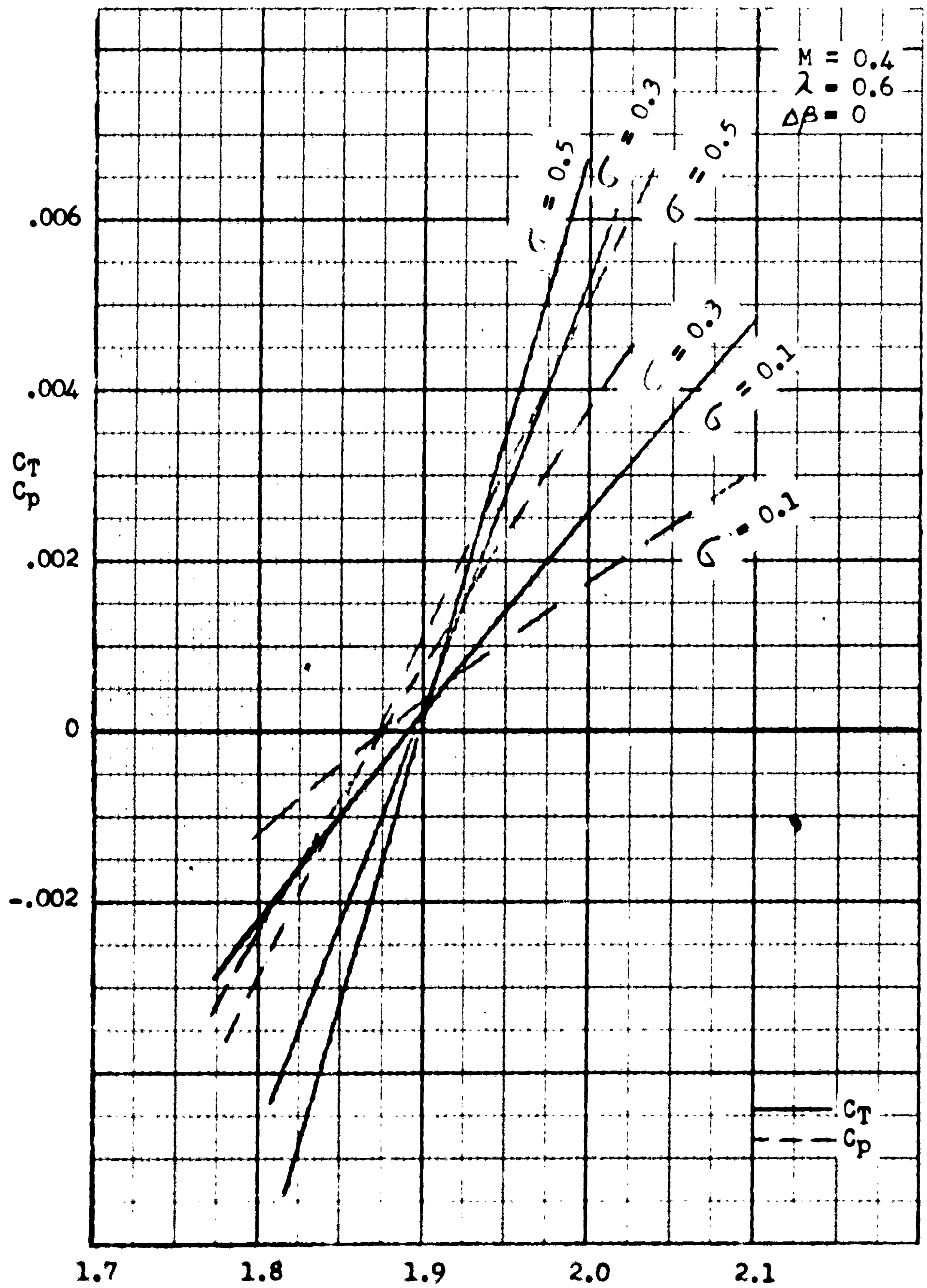
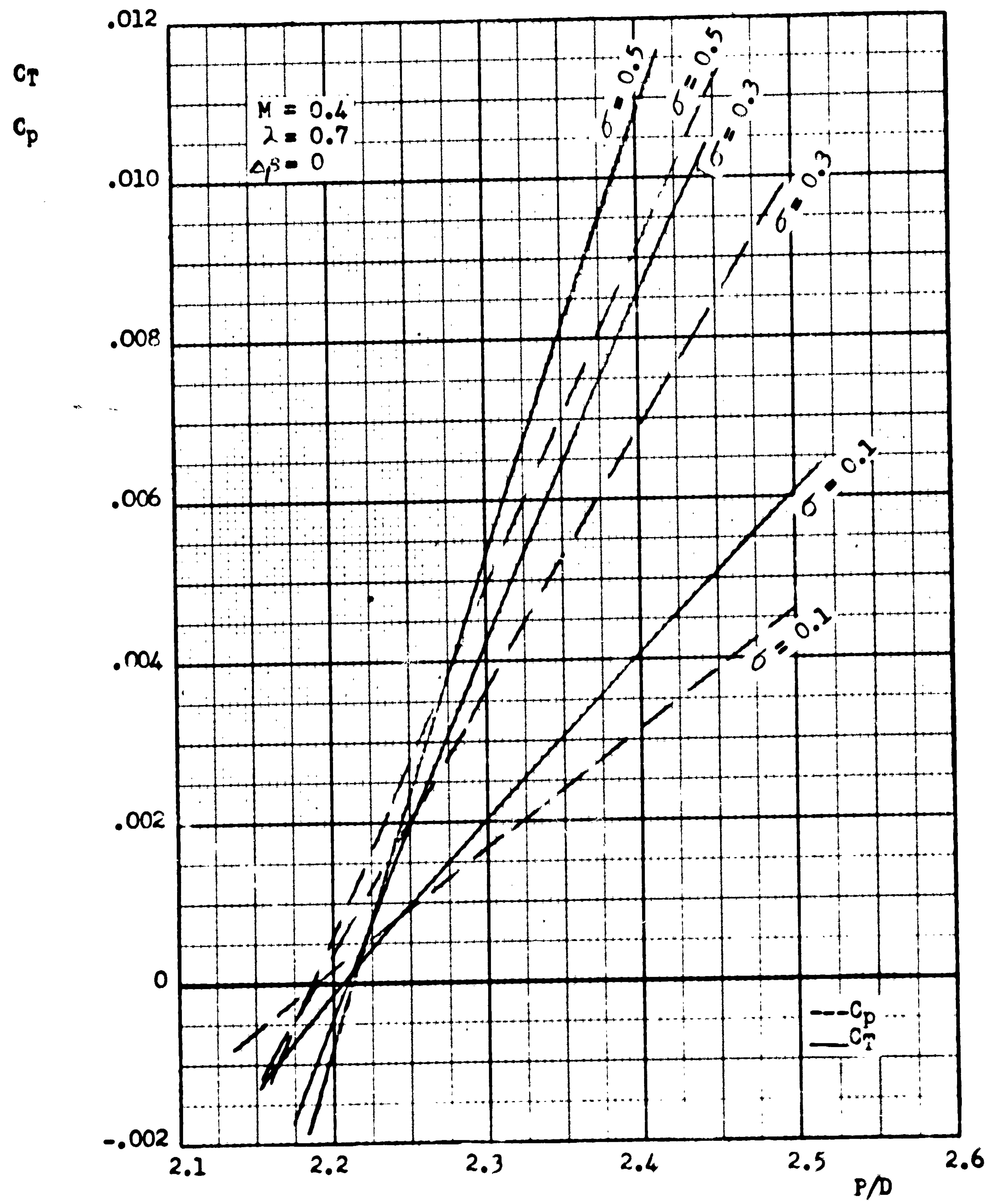


FIGURE 13

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS



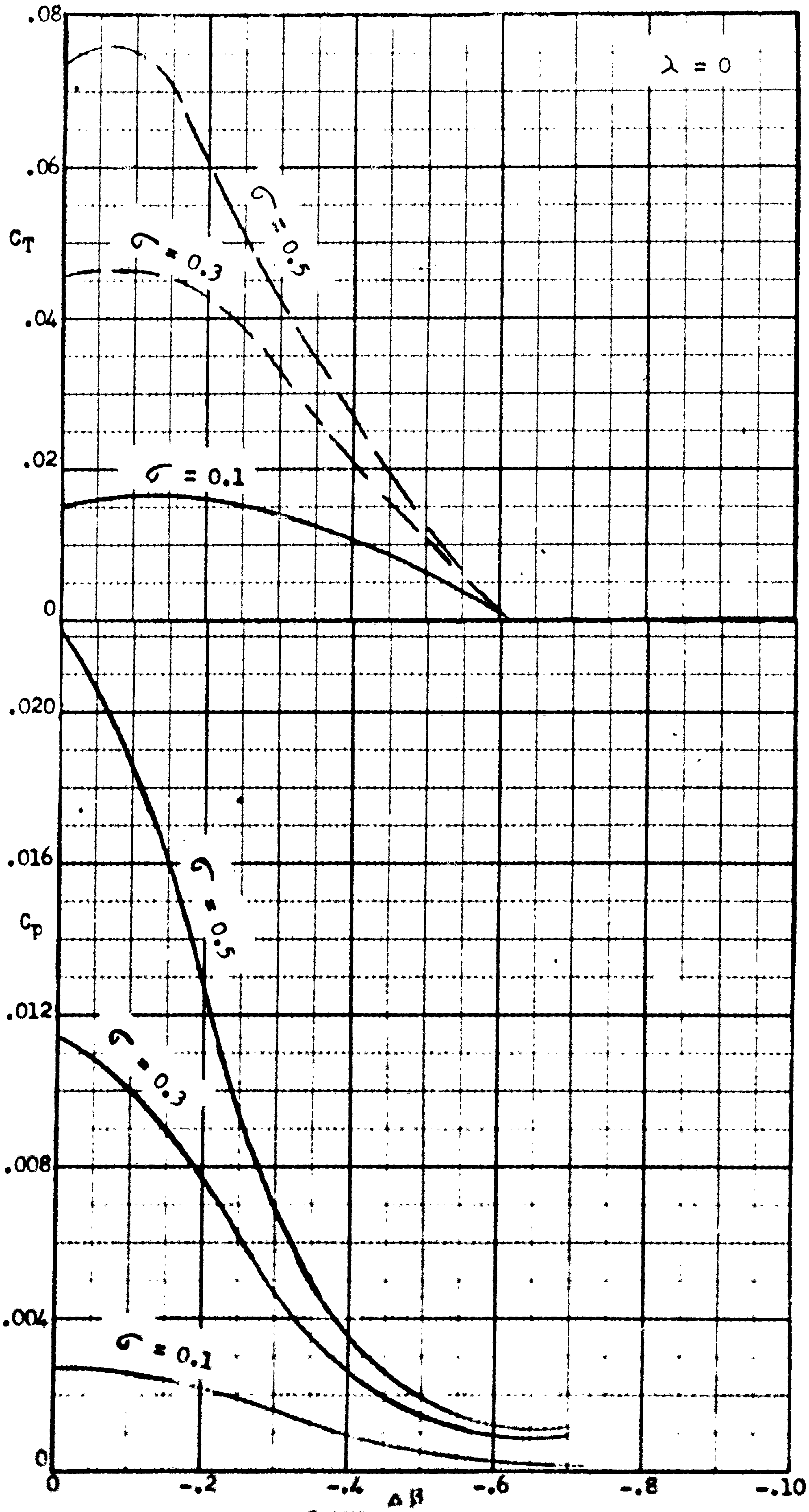


FIGURE 14

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS WITH C_T = .005 ACTING AS ROTORS (λ = 0.5)

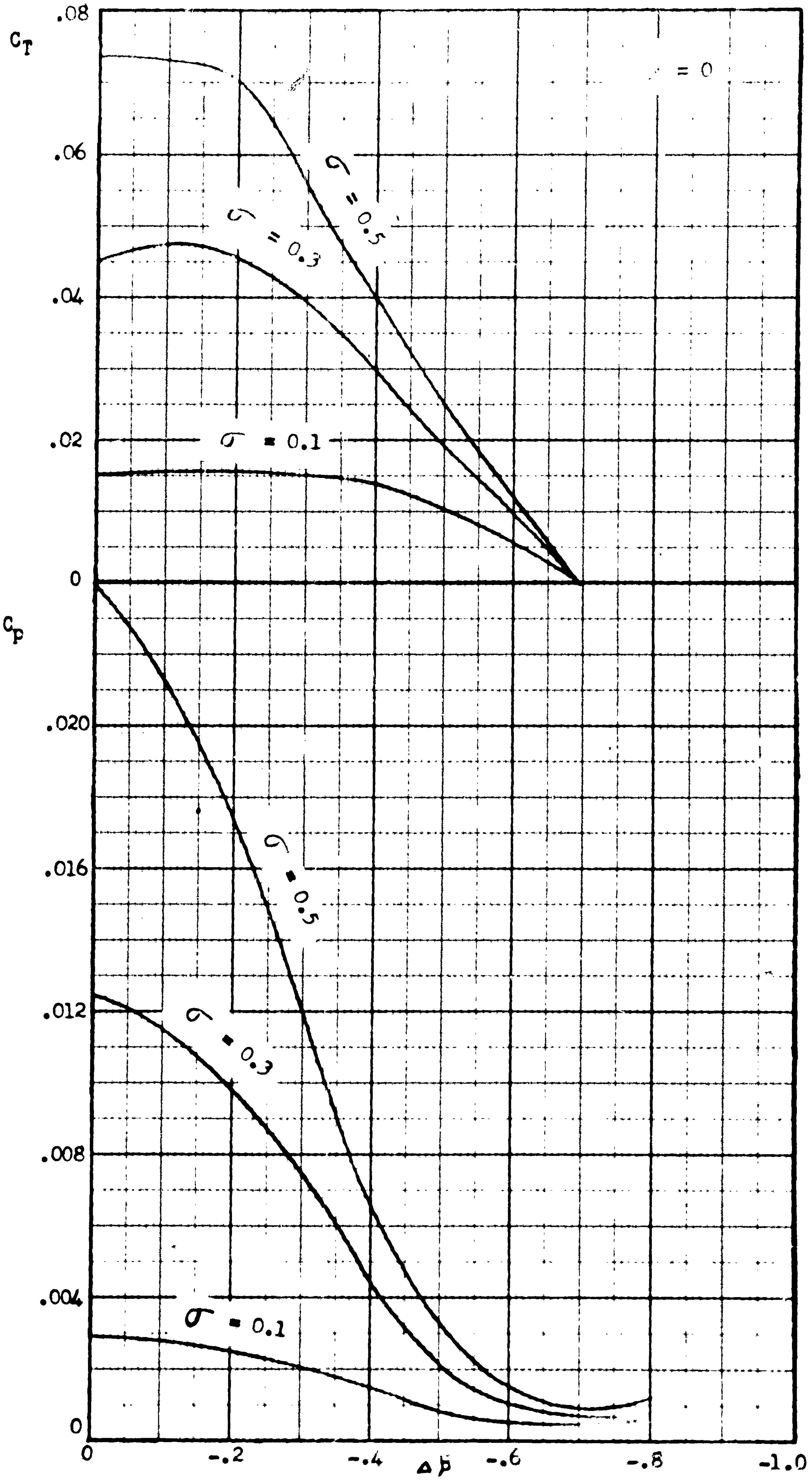


FIGURE 15

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS WITH C_T = .005 ACTING AS ROTORS (λ = 0.6)

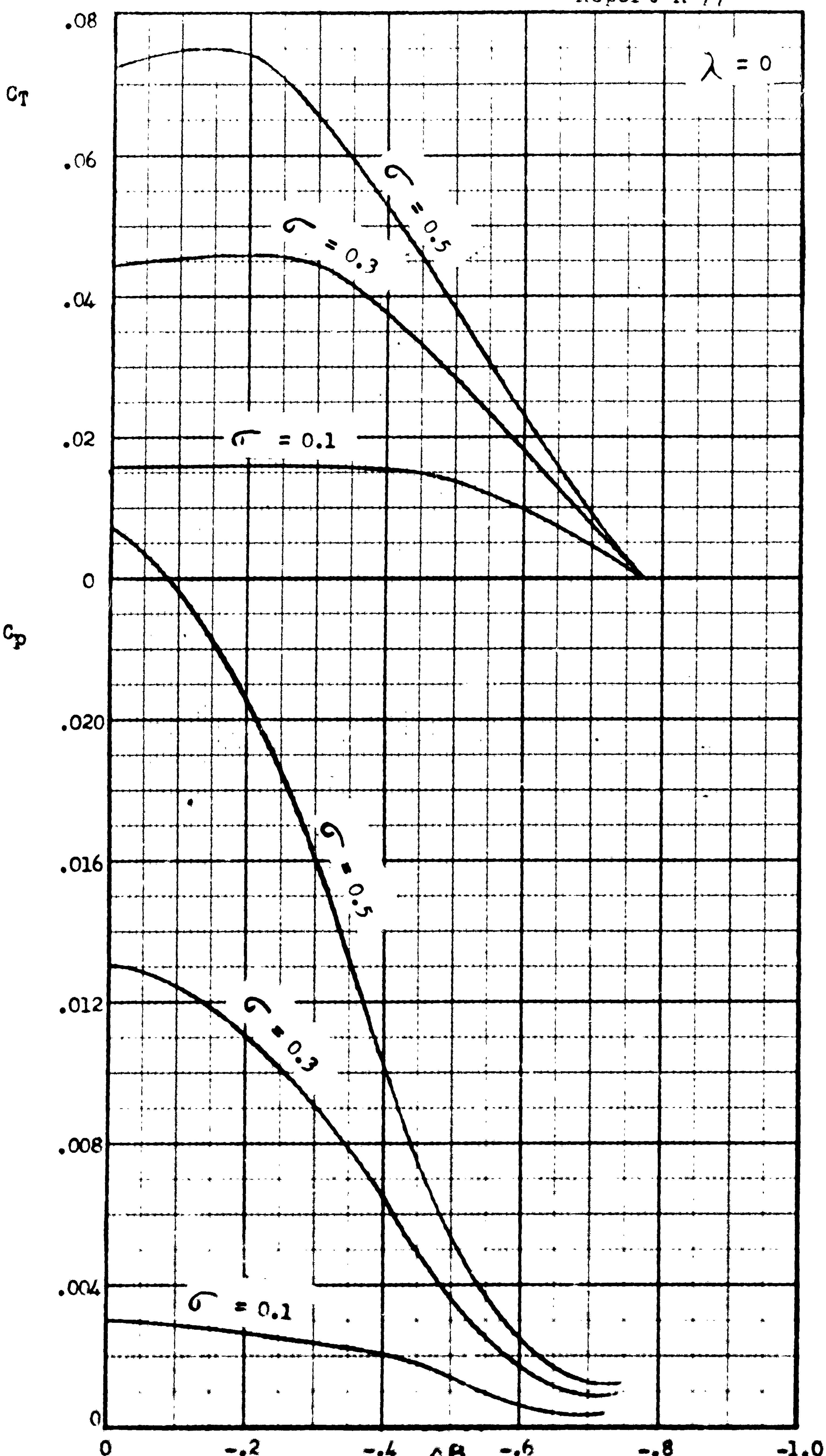


FIGURE 16

CALCULATED PERFORMANCE OF CONSTANT - PITCH PROPELLERS WITH C_T = .005 ACTING AS ROTORS (λ = 0.7)

FIGURE 17

COMPARISON OF POWER REQUIRED BY AN OPTIMUM ROTOR AND AN OPTIMUM PROPELLER WITH BOTH PERFORMING AS A PROPELLER

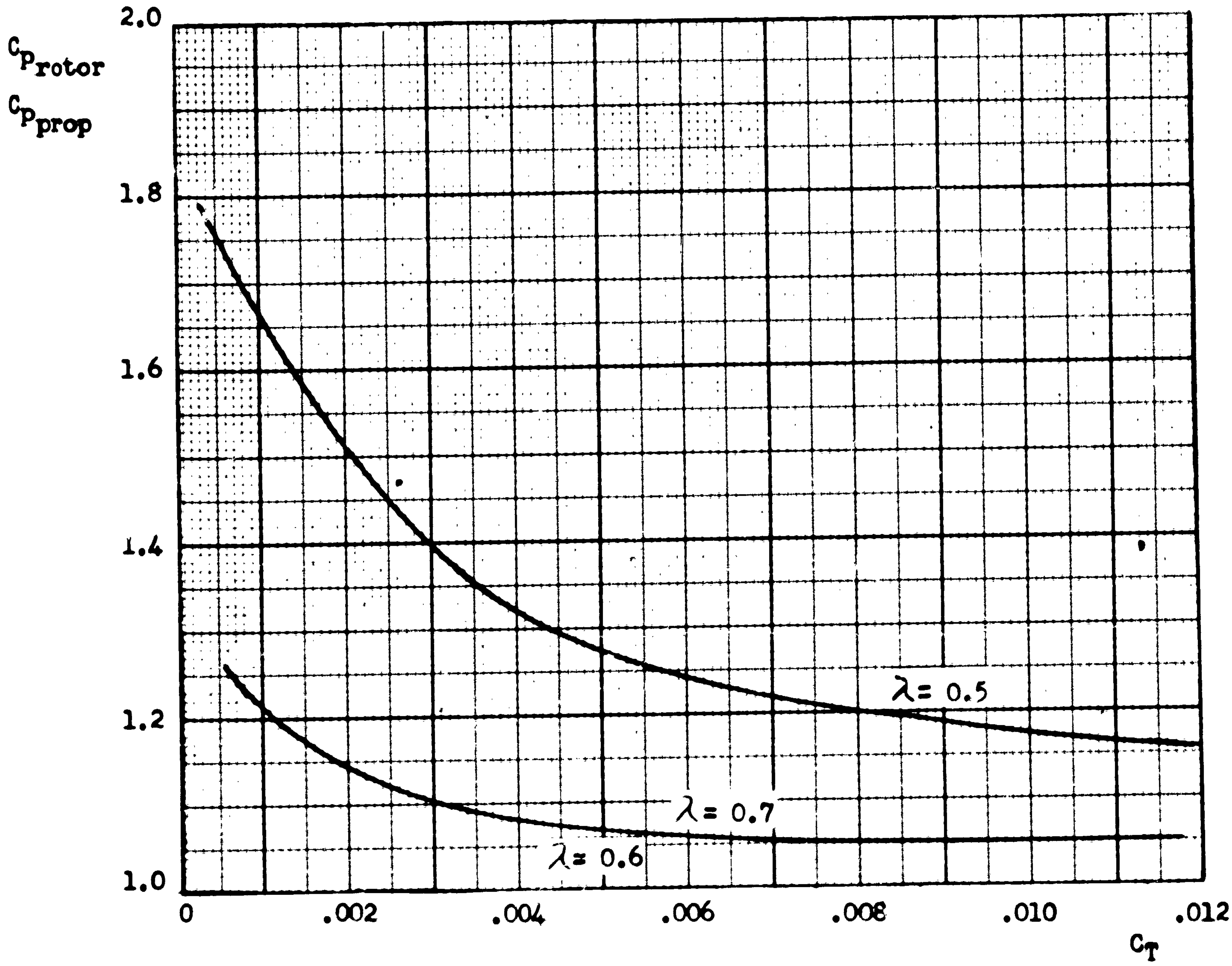


FIGURE 18

COMPARISON OF POWER REQUIRED BY AN OPTIMUM ROTOR AND AN OPTIMUM PROPELLER WITH BOTH PERFORMING AS A ROTOR

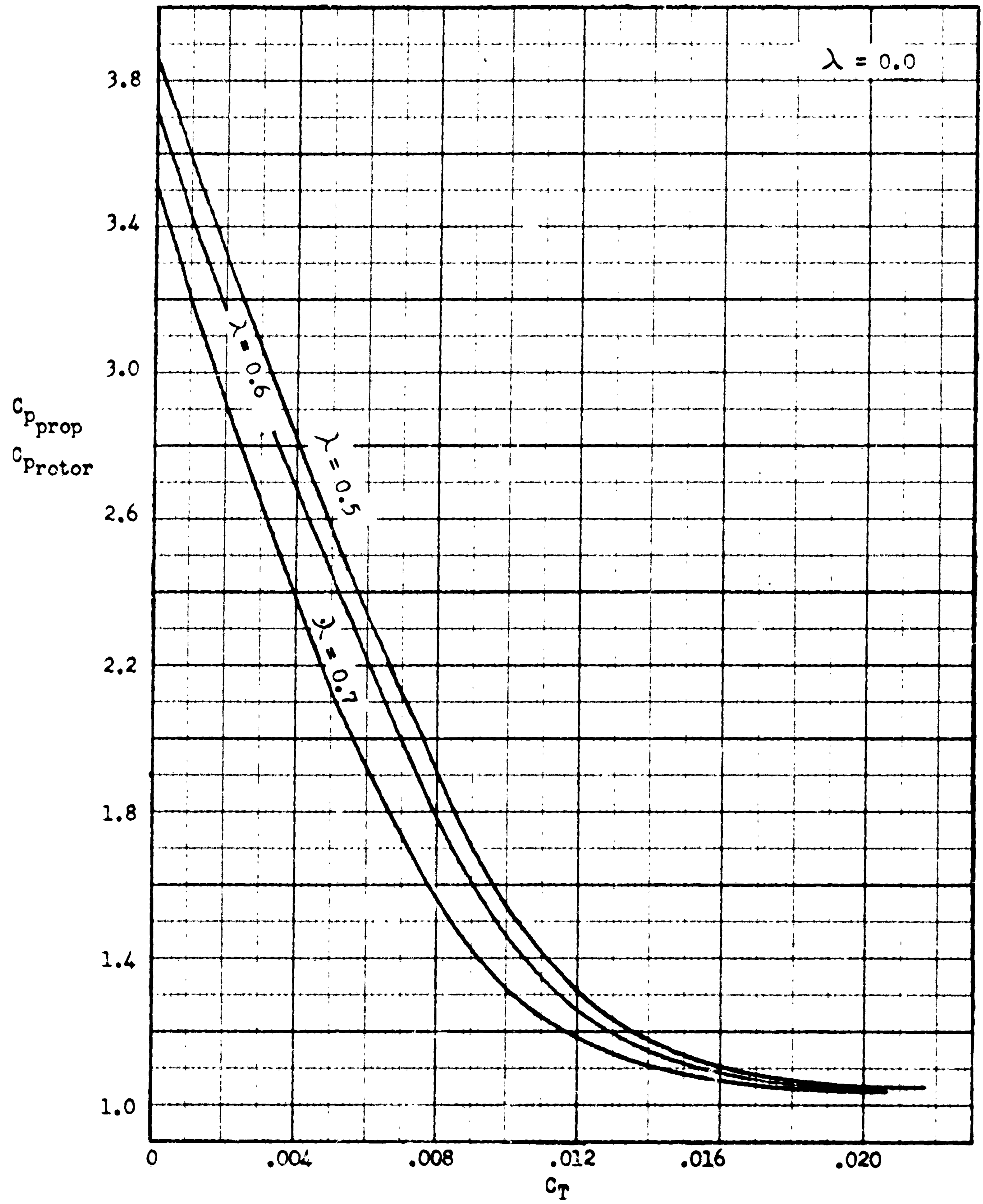


FIGURE 19

POWER REQUIRED BY OPTIMUM ROTOR AND OPTIMUM PROPELLER TO HOVER EXAMPLE TILT-WING TRANSPORT

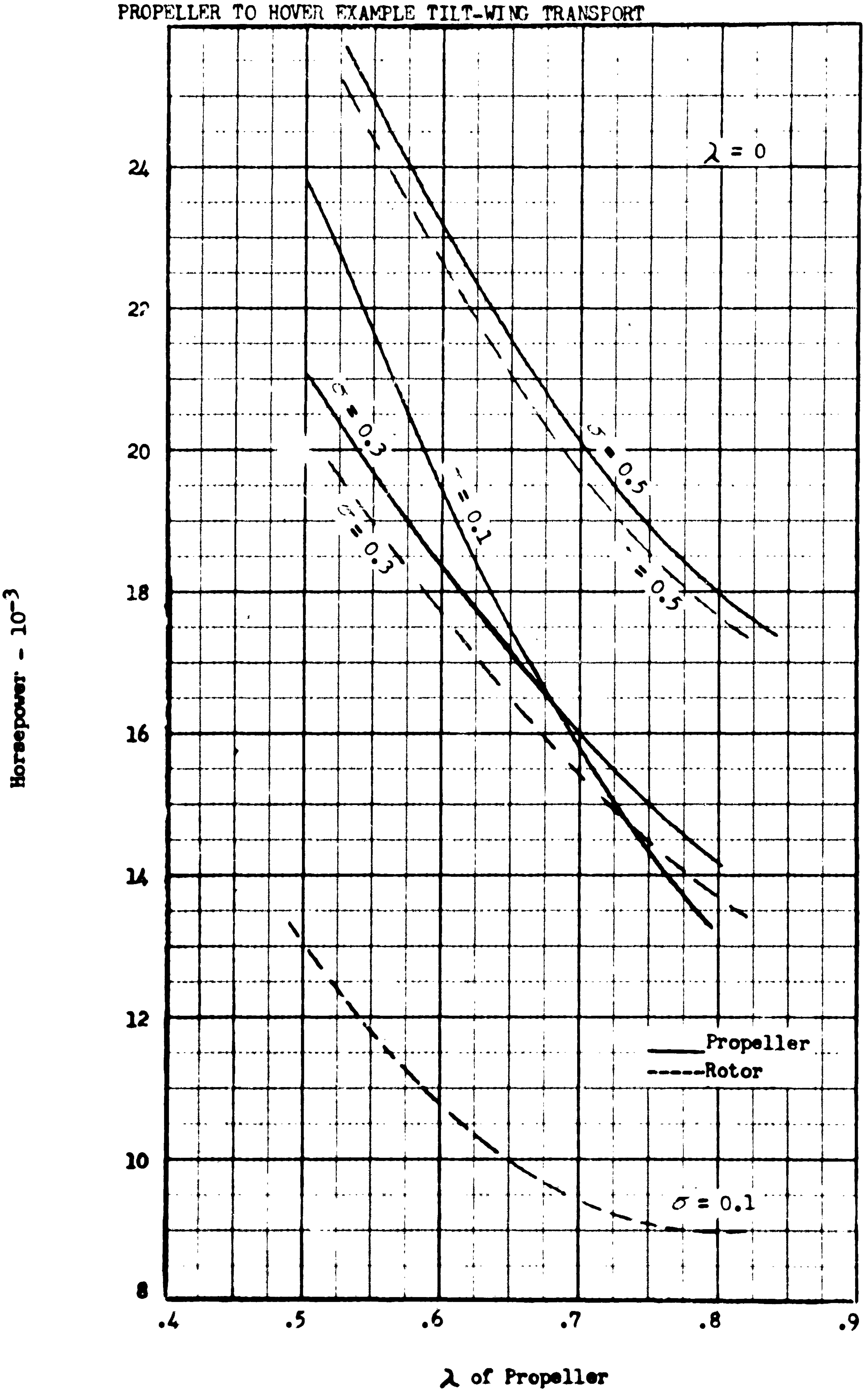


FIGURE 20

POWER REQUIRED BY OPTIMUM ROTOR AND OPTIMUM PROPELLER TO PROPEL THE EXAMPLE TILT-WING TRANSPORT IN FORWARD FLIGHT